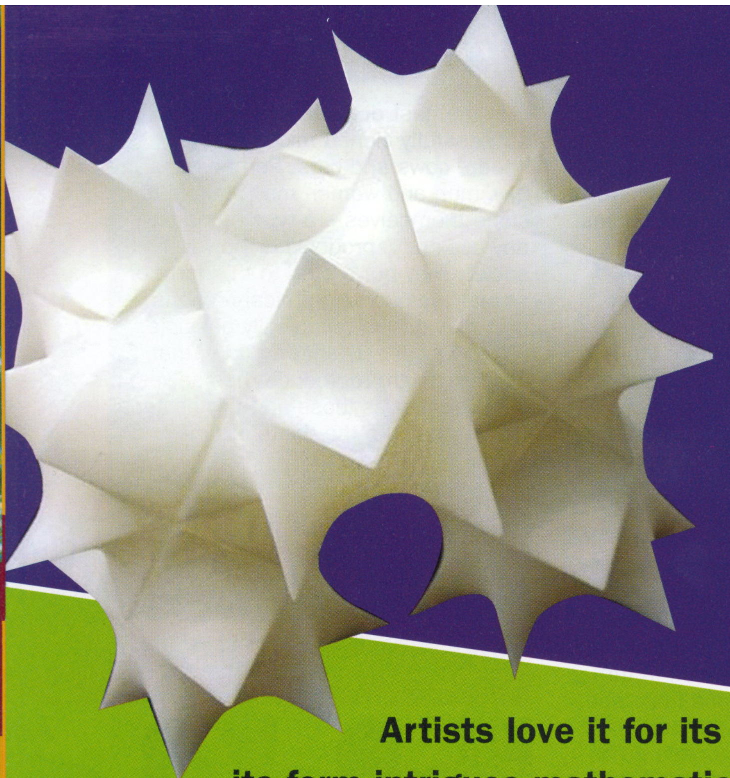
Star
nature's



Artists love it for its beauty; its form intrigues mathematicians; and physicists are using it to understand some of nature's most amazing works.

When you first look at a 'star block', it's hard to believe they could neatly pack together. It is one of a large number of shapes that consist of curved surfaces and straight edges, which can be densely packed together without leaving any space. They were first described by an American architect named Peter Pearce in the 1970s.

In another configuration, star blocks can form an amazingly strong labyrinth where half of the structure contains a network of highly ordered tunnels.

To demonstrate the beauty and elegance of the basic geometry of nature, mathematicians and artists at the Australian National University (ANU), are using innovative techniques to create and analyse the amazing star blocks.

Soap-film surfaces

Each face of a star block is either six-cornered or four-cornered, and corresponds exactly to the surface that a soap-film would form if it was stretched over its edges. If you bend a piece of metal wire into a shape similar to the edges of the star block, and dip it into a 'bubble' mixture, the soap-films would form the surface of the star block.

Soap-film surfaces are a type of minimal surface.

Tim Wetherell

If you were to create a soap-film over wire arranged in the shape of the star block, you would form saddle surfaces that would exactly match the surface of the PD saddle polyhedron.

Figure 1: The star block is a polyhedron with 12 triangular spines. Its surface consists of 10 'saddles' or curved surfaces stretched between the points.

blocks -
fantastic bricks

by David Salt

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Soap-film surfaces are a type of minimal surface – they cover the smallest area within a boundary (using the least amount of material).

Belgian physicist Joseph Plateau was one of the pioneers of minimal surfaces after he began conducting experiments with soap-films in the 19th century.

Today, these surfaces are an important area of research in mathematical and physical sciences because so many of nature's structures use minimal surfaces. Scientists have observed that a minimal surface has to be saddle-shaped at every point (bent in two opposite directions), or flat. If part of a surface is bulging outwards, you can reduce the surface area by replacing the bulge with a flat surface.

But the star block has even more surprises in store.

Stars stack up

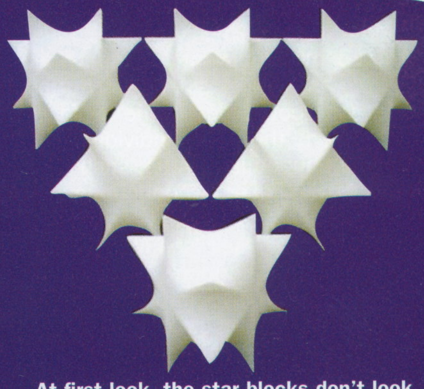
The star block is made up of 10 curved faces or facets, and there are two types of facet. The first type is called a 'primitive saddle' (or P saddle). It's coloured red in figure 1, and has six sharp corners. The other facet is called a 'diamond saddle' (or D saddle). It is coloured brown in figure 1, and has four sharp corners.

The surface of each facet is like a soap-film stretched over the frame, and each block has four P saddles and six D saddles.

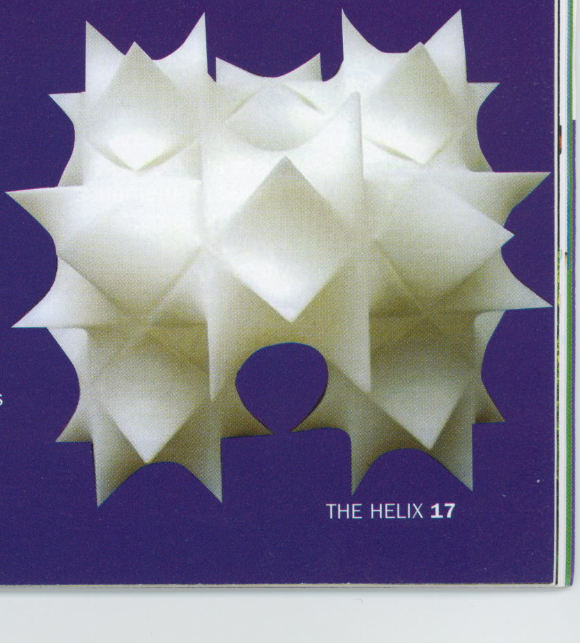
If you stack several star blocks so that the primitive saddles on one block are aligned to join with primitive saddles on other blocks, and likewise for the diamond saddles, you get a seamless match – a pile of blocks with no spaces between them. Mathematicians describe this as tiling space.

Of course, you can densely stack a number of different types of brick shapes – as you do when you build a house with rectangular house bricks. You could do the same with bricks that were triangular or even hexagonal (six corners), in cross-section.

But, bricks with a pentagonal (five-cornered) cross-section can never be stacked without forming spaces in between the blocks. However, unlike the star block, which has faces with a complex curved shape, all these shapes have flat surfaces. But the star block has even more surprises in store. They can be stacked so that only D saddles are aligned to join with D saddles on other blocks, and the P faces are left exposed. This means that, when stacked in this manner, a maze of channels emerges in between the building blocks. What's more, the blocks make up half of the structure and the spaces in between take up an identical volume as the blocks (i.e. 50 per cent is space, 50 per cent is solid block).



At first look, the star blocks don't look very stackable. And yet, match D saddle to D saddle and P saddle to P saddle and they form a seamless whole.



> “The beauty of some of nature’s designs is just stunning.”

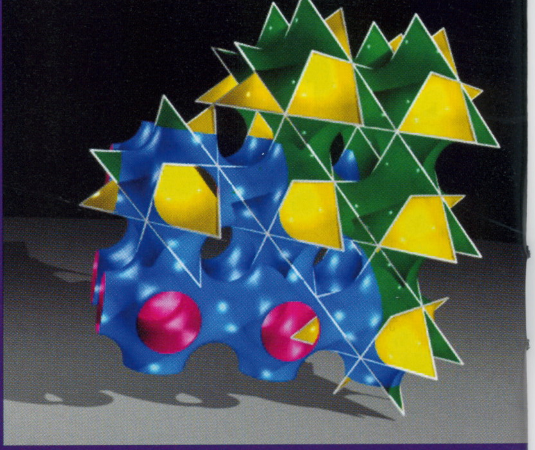


Figure 2: Modelling the crystal lattice of some molecules produced a bi-continuous surface (the pink and blue surfaces in the lower left hand corner of the diagram). The star balls are a breakdown of this surface. The D saddles are yellow, and the P saddles are green.

Amazing mazes

You can visualise this labyrinth of space in figure 2. At the top of the diagram you can see the individual star blocks stacked together. The diamond saddles are yellow and the primitive saddles are green. If you join up all the diamond saddles, the lining of the labyrinth is the exterior surface of the blocks that are left exposed. This surface is formed by the primitive saddles and is referred to as a primitive surface. The primitive space is the blue side of the surface in the picture. However, if you could visualise the inside of the star blocks, it would also form a primitive labyrinth of equal space – shown as the pink side of the surface.

Because both sides of the surface – the blue and the pink – individually form continuous labyrinths, such surfaces are called bi-continuous. One of the great architects of such mazes is nature itself (see *Nature’s art?*).

But wait, there’s more. Guess what happens if you only join primitive saddles and leave the diamond saddles exposed? You get another labyrinth again with 50 per cent of the volume made up of blocks, 50 per cent of the volume in the spaces in between. However, this time the labyrinth is a different shape – a diamond space; but once again another bi-continuous maze.

It is the study of these bi-continuous mazes that led to the rebirth of the star block. To understand how molecules link together to form certain crystal structures, scientists at ANU’s Department of Applied Mathematics have been modelling these mazes. Leading the research is Professor Stephen Hyde, who has demonstrated that many of the properties of a material can be understood by uncovering the geometry of its molecular frameworks.

Gerd Schroeder, one of Stephen’s students, is extending this work by breaking down these bi-continuous mazes into basic bricks – the star block.

From dream to reality

You could build star blocks using metal frames and then stretching heat-shrink plastic over it. This was how the American architect Peter Pearce originally constructed them. However, if you want to experiment with different ways of packing the blocks, you need a number of highly accurate copies of the star block.

So, the ANU scientists turned to Gilbert Riedelbauch, an artist at the Canberra School of Arts who operates the School’s Computer Art Studio, which contains a machine called the ‘rapid prototyper’. This device can turn computer designs into plastic objects that you can hold and examine from every angle.

The rapid prototyper is a computer-controlled robot arm that holds a syringe of molten plastic at one end. As the arm moves around, the syringe ejects a continuous stream of plastic that builds up an object layer by layer.

Professor Stephen Hyde (left) and Gerd Schroeder demonstrate some of the amazing properties of star blocks.

David Salt

As the plastic cools it hardens, leaving you with a three-dimensional copy of your computer design.

Gilbert uses the rapid prototyper to produce amazing hard copies of computer art. It is also used to produce prototype parts and components for researchers and the manufacturing industry.

"The star blocks are amazing," says Gilbert. "However, it's not until you can physically hold them and lock them together in different configurations that their simple genius hits you."

Since constructing the star blocks, Gilbert has also gained an enormous insight on the interplay of mathematics and natural science. "The beauty of some of nature's designs is just stunning," he says. "Before working with Stephen and Gerd I didn't appreciate just how much natural science is devoted to the study of shape in nature."

Building on nature's blocks

The ANU mathematicians now have a precious boxed set of eight star blocks, which they take out whenever someone wants to see how they lock together. Each is around 10 centimeters in diameter.

Yet still, this is not the end of the story. The star blocks are just the first step towards breaking down nature's maze-like surfaces into closed stackable cells. ANU's mathematicians are already looking at how three or more intertwined mazes might be broken down. "We've got mountains of potential blocks that could be worked up," says Stephen. "The scope of this work is breathtaking."

And, as if this isn't enough, the star block itself may be developed further. "Rapid prototyping has given us our first model," says Gerd. "Now it might be possible to use this prototype to cast the block in different materials such as glass or bronze. Builders and engineers have fashioned sky scrapers from plain bricks. Who knows what we might build with a new generation of star blocks. The sky's the limit."

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Stephen, Gerd and Gilbert demonstrate how to put the star blocks together.

David Salt

Nature's art?

Star blocks, and the mazes from which they're derived, are the basis of many of nature's finest structures. The diamond labyrinth, for example, describes the arrangement of carbon atoms in the crystal lattice of a diamond. P and D shapes are also providing us with an understanding of what's happening in many solid and near-solid liquid systems, from hard metal alloys and silicates to polymeric plastics. Even soft materials like oil, water and soap can be coaxed into spontaneously forming these microscopic shapes.

You'll find some of these forms in complicated crystal structures called zeolites. These are natural or synthetic microporous solids, with atomic structures (mostly made of silicon, aluminium and oxygen) that enclose very small, but continuous channel systems, through which water or other chemicals can penetrate. Many processes, from oil refinement to washing powders, make use of the properties of zeolites.

Even the shell of a sea-urchin bears some resemblance to the channel maze constructed from the star block. If you could look at the sea-urchin shell in great detail you'd realise that it is not actually a solid block, but a very porous material with very regular channels running through it. The diameter of the channels is approximately five micrometres – roughly 10 times thinner than a human hair.

The star block is proving valuable in understanding natural processes, which comes as no surprise to Stephen. "Any nice mathematical object ends up being important," he says. "It's the way the world works."